Polytope–Tableau Correspondences for Type A KR Crystals

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Abstract

In this talk we study the polytope model of the Kirillov–Reshetikhin (KR) crystal $KR^{i,m}$ for type A. We give an inductive formula for producing a path from the highest weight element to any given vertex in the crystal graph. For $i \leq 2$ or $i \geq n-1$, we determine the explicit image of any element under the affine crystal isomorphisms between the polytope and the tableau realizations of the KR crystals.

Definition 1. *The set $B_n^{i,m}$ along with the usual crystal operators e_ℓ, f_ℓ and crystal functions $\epsilon_\ell, \varphi_\ell$ for $\ell \in \{i, i+1, \ldots, n\}$ gives the polytope model of KR crystal $KR^{i,m}$ for type A.*

Our main result is the following.

Theorem 1. *For $A \in B_n^{i,m}$, the operator $\mathcal{K}^{A^{(r^A)}} \cdots \mathcal{K}^{A^{(n)}}$ sends the highest weight element to A, where*

$$\mathcal{K}^{A^{(k)}} = \Big(\prod_{j=i+1}^{k} f_{k+i+1-j}^{\varphi_{k+i+1-j}} \Big) \Big(\prod_{j=t^{A^{(k)}}+1}^{i} e_{i+t^{A^{(k)}}+1-j}^{h(k)} \Big) \Big(f_t^{g(k)} \prod_{j=t^{A^{(k)}}+1}^{i} f_j^{\varphi_j} \Big),$$

and $r^A, t^A, A^{(k)}, h(k), g(k)$ are appropriately defined for $r^A \leq k \leq n$.

Keywords: KR crystals, crystal graphs, polytope model, Young tableaux
MSC: 05E10, 17B37, 05E05

References

[1] Dipnit Biswas, Irfan Habib, On the Polytope Model and Near End Node Isomorphisms of Type A Kirillov–Reshetikhin Crystals, arXiv:2508.19796, 2025.